OPEN DISCUSSION

Servadio: Thank you very much, Dr. Bleksley. Is there anybody who has questions to ask?

Burke: This would apply to both papers. To what extent does the tautological factor operate here? As I recall, Caen tried to prove that mathematics was synthetic knowledge, but I think that most theorists of the subject agree that it's analytic knowledge, that is, it's just carrying out the implications of the term. Now for instance, obviously, when the computer worked out π to a thousand degrees, we're not going to give it a psi factor. It was simply carrying out the implications of a terminology. You set up a terminology, and implicit in that terminology are certain kinds of implications, and the great moment of discovery is when you look further into those implications. So, in a sense, it does become like the "Platonic heaven." That is, if those implications are already there, given those terms, those possibilities were already there once you set up that terminology, and then you would even find it in Euclid. All these propositions were implicit in the very kind of approach that he had to the subject, and it occurs to me this is the essential element, as I'm always stressing the symbolism of these matters and the implications of those symbols.

Servadio: Dr. Bleksley, please.

Bleksley: Mr. Chairman, I think this was really what I was trying to get at right from the beginning. The point is there are certain things that are not forced upon us from the outside. Caen believed that the basic assumptions of the Euclidean geometry were forced upon us by the outside world. We now know he was wrong, because a better fit to the outside world is not found in the assumptions of Euclidean geometry, but it took people a long time to find the others because the Euclidean assumptions are the easy ones. Once you have made your body of assumptions, once you've established your set of axioms, then you have called into existence things which you cannot anticipate and which must be explored in great detail. Now this is undoubtedly a tautological attitude, but there is nothing compulsive about your basic assumptions. There are lots of examples of this kind of thing. A very good example is given by the history of what are known as quaternions. Everybody knows that seven times eight is the same thing as eight times seven, but the discovery of quaternions in the early nineteenth century by William Rowan Hamilton had enormous implications for physics and for mathematics subsequently. It couldn't come about until one day while he was crossing a bridge on a walk with his wife, Hamilton suddenly realized that he would not progress as long as he insisted without thinking about it that seven times eight must be the same as eight times seven. This is true for ordinary numbers, but he was trying to invent a new mathematical rule that no one had ever thought of before, and he found that in order to succeed in doing what he wanted, he had to make the rule that A times B is not B times A, but minus B times A. In other words, he had to take a rule which we normally accept, which we call the commutative law (ordinary algebra), and change it. The moment he did that, a new mathematics came into existence which physicists use and in fact are still using to describe mathematically such things as the rotation of a top. If you want to describe mathematically the spinning of a top, one of the ways of doing it is to use a mathematics in which seven times eight is not eight times seven. But you see now that part of it was invention. That was not tautological. Once that step had been taken, what followed was tautology, if you like. It was exploration and discovery, not invention.

SERVADIO: Dr. Margenau wanted to add something.

MARGENAU: I want to add the following, which is meant to be a direct answer to your question as to the tautological aspects of ideas in mathematics. May I recall to you the distinction in words I made. We have the facts of the world which are unrational and against these we have the ideas, the constructs which are entirely at our rational control, at our disposal. We can do anything we please with them, but they stand in rational connections. They are related logically, mathematically. Now, there are two kinds of science: the applied sciences and the pure sciences. In the applied science, we have on the one hand the primary facts and on the other hand the constructs which are regulated by logical procedures. These are not enough to establish a science. Remember I said that in order to correlate the two, we need rules of correspondence, a primary example of which is Bridgman's Operational Definition. Now, an applied science like physics or mathematics or chemistry has both fields, the *P* field and the *C* field plus rules of correspondence which link immediate observations to theoretical terms. That's a picture of a complete applied science. Now the purely formal sciences, pure mathematics and logic, do not need a *P* field at all. They proceed with their own devices. So pure mathematics doesn't need the external experience which applied science demands. For example, there was something called a Hillward Space Object. This was a beautiful theory, most appealing because of the elegance of its structure, the immensity of the conclusions, the theorems to which it gave rise. But for a long time there was absolutely no rule of correspondence linking the items of that algebra with facts. Along came people like Eisenbach, who showed that there are, in fact, such rules of correspondence, and Hillward's space now corresponds to the observed states of electrons and atoms, etc. You started here with a purely intellectual, irrational tautological artifact which is beautiful and consistent. And all of a sudden somebody discovered rules of correspondence between that and the facts in the world, and now you have an applied science. So you have both. Now you wonder about the connection between words and things. I spoke about Bridgman's Operational Definition; then I said there's a more genuine class of correlations called rules of correspondence. Now the relation between things and words is one of the rules of correspondence. It operates precisely in the same way that the Operational Definition does. It links facts or things with ideas, so that's just another illustration. As long as you are dealing with facts, with things and words, you are playing around with an applied science, not a pure science.

BURKE: The one thing they made of Bridgman is that you cannot give an operational definition of operationalism.

MARGENAU: This is imposed by the necessity inherent in the construction and observations must obey the tautologies of the *C* field. Now so far as Bridgman is concerned, he has indeed been criticized and stands in need of criticism because of his insistence that all definitions must be operational. We now recognize many kinds of operations. Operational definitions are necessary for things that aren't measured, or what we call observed, but ideas in physics, in chemistry are not measured, any more than the ideas of desire or love.

JENNINGS: I'm fascinated with both the mathematician and the physicist referring to this slit through which you monitor all experience in the world itself, and you, Mr. Burke, are hinting at that strange filter that poets use when they look at the universe. It seems to me that we're all talking about the "selective eye," or "selective ear" that we use when we look at our world. There is a hypnagogic experiment in which you send a man into a room to look around, come out, and name the number of objects he can see, and they're quite a few. And then under hypnosis, you send him back again and he comes back reporting hundreds of items. If we were able to record, if we were able to deal with all that we do in fact record, we'd go mad, and it's that filter that we've all been talking about that makes it possible for us to deal with the universe. Could it possibly be that this psi factor as you're describing it, Professor Margenau, is in fact this highly sensitive narrowing or widening of that receptive slit?

MARGENAU: Precisely.

MANGIONE: As a pure non-scientist, I would like to raise this very amateur question. We've been talking about the mystery of the psi functions and sometimes in a very mysterious way I've heard the word "Divinity" come into it, which seems to me a method of abdication. But I wonder if we know enough about the thought process so that perhaps we could start dissecting a non-thought process—whether there is, perhaps, in some biological way an indication of particles inside the human system that might give us some insight into the psi factor?

WALTER: I think it was Spinoza who said that "consciousness is the idea of ideas." I think this notion of a regress may be helpful here. What you're rather suggesting in many of these presentations this morning here is a sort of regress, not necessarily infinite regress. As a matter of fact, I would suggest that one can have infinite regress of the paranormal nerve problem by remembering the amount of information that is lost in each reflection into vagueness. But I suggest that thinking may be something like the idea of ideas which could have perhaps two or three regressions, but not many more. There is a great difficulty in the infinite regress and the experimental scientist withdraws from this with revulsion because it's an impossible situation. But I suggest that the idea of ideas and possibly one more stage, the idea of the idea of ideas is a rational concept in relation to thinking. There are, in other words, levels or systems of mutual reflection. As was said this morning, for example, between artificial systems (such as mathematics) and cerebral or natural ones, is this constant mapping or matching of the outside world (the world of senses) against constructs or models, and a detection of match or mismatch between these. This is a highly dynamic process in a sense that one tries to match one thing with the other, but when this match occurs, this may be an inspiration, not simply a logical process or deduction. This allows for induction and not simply induction by enumeration, which is also a rather trivial process, but induction by guessing, in effect, by intuition. This is human philosophy, and in the end nothing can be proved because induction is limited. You can't get any further than this, and so this came to a dead end because it didn't allow for the possibility of guessing or intuition. One builds up big guesses from little guesses, and one is not aware of making a small guess until the big guess comes out, and this differs in all different cultures.

The distinction between discovery and invention seems to me a very important one in all sciences. You might say, for example, that the predecessors of Columbus invented the westward route to the Indies, but Columbus discovered America. The discovery of America depends upon the idea or the invention of the idea that one could circumnavigate the world, and this was essentially an invention. It might not have been true as far as they knew at the time. Then there is the question of the invention of zero. This seems to be a very important step in mathematics. It was the Indo-Arabic culture which invented zero and it seems to me to be a most extraordinary invention, important not only in numerical mathematics but in logical mathematics. One comes to the logical calculus of zero-to-one as in all modern inventions, as for example, in computers. This depends upon the notion of "nothingness." This is not simply "nothing" in the sense that it doesn't matter, but an operational idea. What is the history of this?

SERVADIO: Who wants to say something about "nothingness"?

BLEKSLEY: I'm quite sure that Professor Margenau would be perfectly happy to talk at length on "nothingness," but this is the mathematician's privilege, I think. The point is, of course, that the Hindus didn't invent the idea of "nothing." The idea of nothing is common to all cultures. After all, if you have nothing in your pocket you are thoroughly well aware of this fact. What the Hindus invented was a symbol for nothing, and this, I think, would interest Mr. Burke—because one would feel that since nothing is the absence of anything, you hardly need to identify it by a symbol. But then they discovered that they needed a symbol for "nothing" in order to be able to introduce the ordinary numeration system in which one-naught-naught is a different number from one-naught, and is a different number again from one, without any naughts. In the absence of the naughts, these numbers would have been identical. The Hindus realized that if you wanted to give a one in a number a meaning which depends on the position, you've got to be able to identify the position, so that it can occur in the right-hand digit or in the digit one from the right hand. The two "ones" that occur are not the same nor do they look the same. You write down the number, "one,one,one," which to us is a hundred and ten and one, and those "ones" look alike, but because of their position, they are very different. The right hand one means "one." The second "one" means ten. The third "one" means one hundred. This was, of course, the basic thing that made it possible for mathematics all of a sudden to start flourishing, just at the time when we needed it most—the beginning of the astronomical discoveries of the thirteenth and fourteenth centuries. When computation suddenly came into astronomy, the tools were available. And this was what made it fundamentally important. If it had not been for the discovery of zero by the Hindus in the tenth or eleventh century, astronomy couldn't have existed. The invention of the symbol for zero was not a discovery. Nobody discovered the symbol "naught." You invent a symbol "naught."

JENNINGS: Can you explain why it was that the classical philosophers who had no lack of intuition or imagination failed to appreciate the magic qualities of a positional notation, although they had a very good sense of geometrical space? Why was it not until the Hindus, and then later, the Arabs wrote this, that this notion became so fertile?

BLEKSLEY: I'm not sure that I know why, because this has always been one of the mysteries of mathematical history that the Greeks, who were first class logicians, never saw algebra. They only saw geometry. They regarded arithmetic as a problem for slaves. If Euclid wanted to add up a series of figures, he would never have done it himself. He'd have handed the job over to a slave. Arithmetic was not the queen of the sciences for the Greeks; it was the slave. Geometry was the queen of the sciences. And then all of a sudden, around the tenth and eleventh centuries, this new notation for a number arose, and the moment that became possible, all sorts of other things followed in its train. You could suddenly start analyzing mathematically the movement of the heavenly bodies because you could write down numbers which adequately set this in motion. And astronomy suddenly came in line with that. The astronomy of the ninth century was almost nonexistent. The astronomy of the thirteenth century was a flourishing discipline, and this was mainly due to the intervention of this new computation. But why the Greeks never saw algebra, I don't know. It was simply that one invention which was needed at that time never took place.

MARGENAU: I would like to address myself to Mr. Mangione whose question concerning the ultimacy of explanations in terms of physical particles requires an answer. Let me first call your attention to the fact that there is absolutely nothing in science that prevents the acceptance of an explanation in terms of particles. The physicist will certainly want to know what happens in the brain cell. We want to know what happens when so-called psi takes hold of us. But I believe the physicist will never have the last word because even if he understands the particles in the brain cell in all physiological and psychological processes, he will still not have an explanation. This is the hitch. So I think ultimately you will need correspondence between the two domains.

Now, Dr. Walter, zero is not mere "nothingness." Remember, we have the facts which don't contain zero except as a rather extreme abstraction. On the other hand, we have the constructs in this tautological rational self-consistent team which are linked by the facts through operational definition. Now, zero within the domain of symbols, what I call the *C* field, is not "nothingness." In fact, the idea of zero is absolutely demanded by the principles of consistency and elegance within the analytic domain, so we therefore encounter it as a respectable entity, not as "nothingness." Therefore operation in the *C* field can be matched against the world of facts if you allow this symbol, this entity of zero to correspond. And that's the story. So it does represent nothing on the *P* plane but it certainly is an important entity in the *C* field. I am struck by the analogy between this and the Hindu view of "nothingness." It doesn't mean nothing. It means something, undifferentiated. Concerning the failure of the Greeks to respect arithmetic, I wonder if this may not perhaps be because of the discovery of rational numbers which were like monkey wrenches thrown into arithmetic—could this be a partial answer to the question?

BLEKSLEY: I think this may be so. I hadn't thought of it this way. You remember that part of the Greek philosophy of living was the commensurability of things. The role of a nice regular procession of numbers like one-to-two, two-to-three, and three-to-four, etc., which represent the acceptable sounds, this just disappears as soon as you have the square root of two which can't be written. This is perfectly true. There's proof as provided by Euclid, and it may be that the Greeks felt that this was something that was going to lead them into such bad favor with the gods that they'd better stay away from it.

MARGENAU: It certainly terminated in the atomic theory.

CHU: It is extremely refreshing and inspiring to hear mathematicians and physicists tell us that the human mind can invent as well as discover, because from the human point of view to invent something is value latent, and we all like it and prefer it and welcome it. But may this be a human point of view? To use Dr. Margenau's phrase, we are "moving along a three-dimensional slit along the axis of time." We see things from a certain point of view, and as Mr. Jennings has pointed out, it is selective. Mr. Mangione has asked a question of biological basis. It may be that we are biologically constructed in such a way that we must see the universe in a selective way, wider or narrower, and in doing so, we think that we have seen something that never existed before and that we have invented. Let me give the very crude analogy of an hourglass. The sands from the upper chamber come down to the lower chamber through a bottleneck. One grain at a time and no more. Perhaps our human mind is at the bottleneck and we see one grain at a time in our consciousness, so to speak, but it may be that that isn't the natural behavior of all sand. Maybe it doesn't go through a bottleneck one grain at a time. So my question is: Can we rule out the possibility that the universe is an undifferentiated whole composed of all potentialities, all possibilities, and the human mind simply invents what it needs and explores all the implications? The human mind is logical and rational, but nature may be non-rational, non-logical.

MARGENAU: The answer is, I think, yes, there is this undifferentiated principle, but I want to make this clear again. You've got to make the distinction between the facts of nature which I call the primary plane of experience, and the logical domain against which we match the facts of nature. The facts of nature do form an undifferentiated whole, and it's precisely because of that that we need the organizing influence of reason, and we do that in the *C* field. Now the theory that I presented this morning has been applied to the idea, although undifferentiation continues. The *P* plane, which has no order, is almost undifferentiated, and that's what we call nature. On the other hand, the ideas in their rational context are indeed organized and differentiated and that's exactly what science does. It acknowledges the existence of this undifferentiated whole and organizes it by principles. Now the question is, which do you want to call real?

CHU: But the question is could it be that the constructs themselves are also included in the undifferentiated whole?

MARGENAU: I suppose so. But I would still maintain that the distinction is an important one.

GADDINI: When we speak of theory of science, we could also discuss what is a theory? A theory in science? We may try to compose in our theorizing and maybe different theories will develop in the future according to different theoretical directions that appear. I get a little uneasy at the end of these discussions because I always think that we should talk about psi factors. There seems to be no contact, no continuity between science and psi factors. If you theorize in science, the moment you find yourself in front of psi factors, then you have just admitted something that goes beyond. We have to then turn back and re-examine what we left behind years ago. If we look at things this way, we may perhaps conceptualize psi factors as still part of the biological foundations. Psi factors, in a way, are the cosmic world in ourselves, part of our body. The more I think about psi factors, the more I visualize them as part of our body contact with the rest of the world, a kind of taking part in reality which we cannot do any more under the plan of the intellectual theory, but that some part of us can still do in the body way which we lost billions of years ago but which is different from us, is something from without.